

# Limitations of the Hecht Equation Encountered in Measuring $\mu\tau$ Products in Mercuric Iodide

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## ABSTRACT

While attempting to measure  $\mu\tau$  products in HgI<sub>2</sub> using the pulse rise-time method, a method based on the Hecht equation, fundamental limitations of the Hecht equation were revealed. These limitations restrict the equations utility, even for the case for which it was originally developed, i.e. when carrier trapping occurs without de-trapping. The Hecht equation purportedly analytically characterizes the charge collection process under uniform field and permanent, charged carrier trapping conditions. Since de-trapping occurs in HgI<sub>2</sub>, particularly in high spectral performance HgI<sub>2</sub> detectors, it was recognized that limitations would be encountered in applying the Hecht equation.

However, several limitations exist that are intrinsic to the Hecht equation, which are associated with the two primary and independent terms of the equation: 1. The charge collection efficiency term  $Q_0\{t/t_r\}$ , which depends on the ratio of the carrier lifetime or trapping time to the transit time; and 2. The asymptotic, negative exponential carrier lifetime or trapping time dependent term,  $[1 - \exp(-t/t)]$ .

The dependency of the first term of the Hecht equation on the ratio of the carrier lifetime to the transit time creates the anomalies that for times less than the transit time full charge collection can never occur and the maximum achievable charge collection value increases monotonically with carrier lifetime. In addition, for carrier lifetimes greater than the transit time the Hecht equation predicts full charge collection always is achieved at long charge collection times. These constraints or conditions do not necessarily represent physical reality, but are anomalous features intrinsic to the equation.

Through substitution for the transit time, the first term of the Hecht equation can be converted so as to depend upon the carrier mean free path length  $\lambda$ , an intrinsic material transport property parameter, and on the detector thickness  $d$ , an extrinsic detector parameter: i.e.  $Q_0\{t/t_r\} = Q_0\{L/d\}$

This form of the first term of the Hecht equation creates the anomalies that for carrier mean free path lengths less than the detector thickness the equation predicts full charge collection can never occur and the maximum achievable charge collection value increases monotonically with carrier mean free path length.

Another anomaly or irregularity is intrinsic to the second term of the Hecht equation,  $[1 - \exp(-t/t)]$ . The reciprocal exponential dependency on carrier lifetime or trapping time, in conjunction with the asymptotic structure (i.e. subtraction from 1), result in the anomaly that equivalent charge collection times increase with increasing carrier lifetimes. This phenomenological trend is opposite to intuition and to experimental observation. When combined the effects of these anomalies from both terms of the equation result in predicted charge collection behaviors that radically depart from those observed with actual detectors.

Keywords: mercuric iodide, Hecht equation,  $\mu\tau$  product; mobility, carrier lifetime, trapping time, de-trapping time, transport properties, room temperature semiconducting radiation detectors

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## 1. INTRODUCTION

The major focus of the present development of mercuric iodide (HgI<sub>2</sub>) as a radiation detector material at Constellation Technology Corporation (CTC) centers on hole transport properties. Hole transport is characterized by the mean free hole path length  $\lambda$ . This property determines the detector thickness capability, detector yields and spectral resolution performance of planar, high energy, HgI<sub>2</sub> gamma-ray detectors.

The mean free path length is the distance the carrier travels at carrier velocity  $v$  for a lifetime or trapping time  $\tau$ .

$$l = vt \quad (1)$$

In the linear regime the carrier velocity is proportional to the electric field  $E$  via the proportionality constant, i.e. the mobility  $\mu$ . The electric field is equated to the applied voltage bias across the detector thickness  $d$ . The transit time  $t_r$  is the time for carrier transit across the detector thickness  $d$  assuming that no trapping occurs.

$$v = \mu E = \mu V_b / d = d / t_r \quad (2)$$

Substituting for the velocity from equation 2. into equation 1. one obtains the carrier mean free path equated to the intrinsic material parameters for carrier transport, i.e. the product of mobility  $\mu$  and the lifetime  $\tau$ , and the extrinsic detector parameters, i.e. the thickness  $d$  and the applied bias voltage  $V_b$ .

$$l = \mu \tau E = \mu \tau V_b / d \quad (3)$$

From this formulation for the mean free carrier path length it is apparent as to why the measurement of the  $\mu\tau$  product characterizes the material carrier transport properties. Materials research and detector fabrication refinement efforts are ongoing at Constellation Technology to improve the four primary material and detector properties that affect hole transport, i.e. hole mobility ( $\mu_h$ ) hole lifetime or trapping time ( $\tau_h$ ), hole de-trapping time ( $\tau_h^d$ ) and maximum achievable applied bias voltage ( $V_b$ ).

The Hecht equation was developed to analytically characterize the charge collection process in semiconductor radiation detectors under uniform field and permanent, charged carrier trapping conditions (1-4). The Hecht equation is comprised of two primary terms: the charge collection efficiency term  $Q_0\{t/t_r\}$  ( $t_r$  = transit time) and the asymptotic, negative exponential carrier lifetime or trapping time and elapsed time dependent term,  $[1 - \exp(-t/t)]$ .

$$Q(t) = Q_0(t/t_r)[1 - \exp(-t/t)] \quad (4)$$

While attempting to measure  $\mu\tau$  products in HgI<sub>2</sub> using the pulse rise-time method, a method based on the Hecht equation, limitations of the Hecht equation were encountered. These limitations restrict the equations utility, even for the case for which it was originally developed, i.e. when carrier trapping occurs without de-trapping. These anomalies are in addition to those associated with de-trapping in HgI<sub>2</sub>, particularly observable in high spectral performance HgI<sub>2</sub> detectors.

## 2. ANALYSIS & DISCUSSION

The limitations intrinsic to the Hecht equation are associated with the two primary and independent terms of the equation i.e. the charge collection efficiency term  $Q_0\{t/t_r\}$  and the asymptotic, negative exponential carrier lifetime and elapsed time dependent term  $[1 - \exp(-t/t)]$ .

The dependency of the first term of the Hecht equation on the ratio of the carrier lifetime to the transit time creates the anomalies that for times less than the transit time full charge collection can never occur and the maximum achievable charge collection value increases monotonically with carrier lifetime. These anomalies are evident in the graphs of the Hecht equation presented in Figures 1-3. In addition, for carrier lifetimes greater than the transit time the Hecht equation predicts full charge collection always is achieved at long charge collection times. These constraints or conditions do not necessarily reflect reality, but are anomalous features intrinsic to the equation.

Through substitution for the transit time from equation 1., the first term of the Hecht equation can be converted so as to depend upon the intrinsic material transport property parameters (i.e. the carrier lifetime  $\tau$  and the carrier mobility  $\mu$ ) and on extrinsic detector parameters (i.e. detector thickness (i.e. geometry) and the applied bias voltage  $V_b$ ).

$$Q_0\{t/t_r\} = Q_0\{tv/d\} = Q_0\{tmE/d\} = Q_0\{tmV_b/d^2\} \quad (5)$$

Recognizing that the product of the carrier lifetime and the carrier velocity is the carrier mean free path length ( i.e. the intrinsic material transport property parameter), a further substitution converts this first term so as to depend upon the mean free path length and the detector thickness.

$$Q_0\{tv/d\} = Q_0\{tmE/d\} = Q_0\{l/d\} \quad (6)$$

This form of the first term of the Hecht equation creates the anomalies that for carrier mean free path lengths less than the detector thickness the equation predicts full charge collection can never occur and the maximum achievable charge collection value increases monotonically with carrier mean free path length.

Another anomaly or irregularity is intrinsic to the second term of the Hecht equation,  $[1 - \exp(-t/t)]$  (see Figure 4). The reciprocal exponential dependency on carrier lifetime or trapping time, in conjunction with the asymptotic structure (i.e. subtraction from 1), result in the anomaly that equivalent charge collection times increase with increasing carrier lifetimes. This phenomenological trend is opposite to intuition and to experimental observation. When combined the effects of these anomalies from both terms of the equation result in predicted charge collection behaviors that radically depart from those observed with actual detectors (see Figure 1,2 & 5).

## ACKNOWLEDGEMENTS

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Figure 1. Hecht Equation. Hole Transport.  $\mu = 4 \text{ cm}^2/(\text{V}\cdot\text{sec})$ .  
 $t_r = 5\text{E-}6 \text{ sec}$ ;  $V_b = 2000 \text{ V}$ ;  $d = 2\text{mm}$

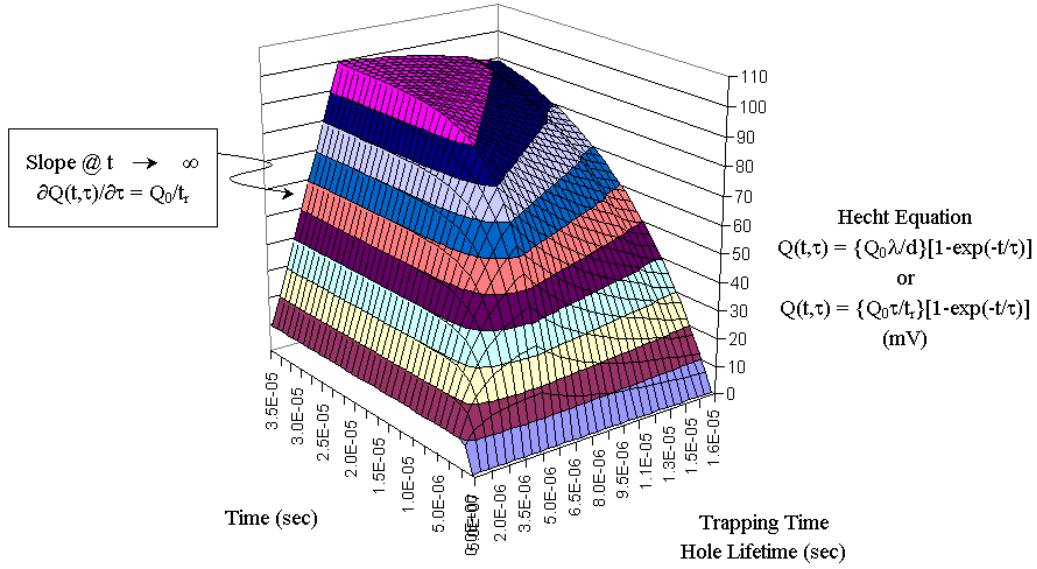


Figure 2. Hecht Equation. Hole Transport.  $\mu = 4 \text{ cm}^2/(\text{V}\cdot\text{sec})$ .  
 $t_r = 7.5\text{E-}6 \text{ sec}$ ;  $V_b = 3000 \text{ V}$ ;  $d = 3\text{mm}$

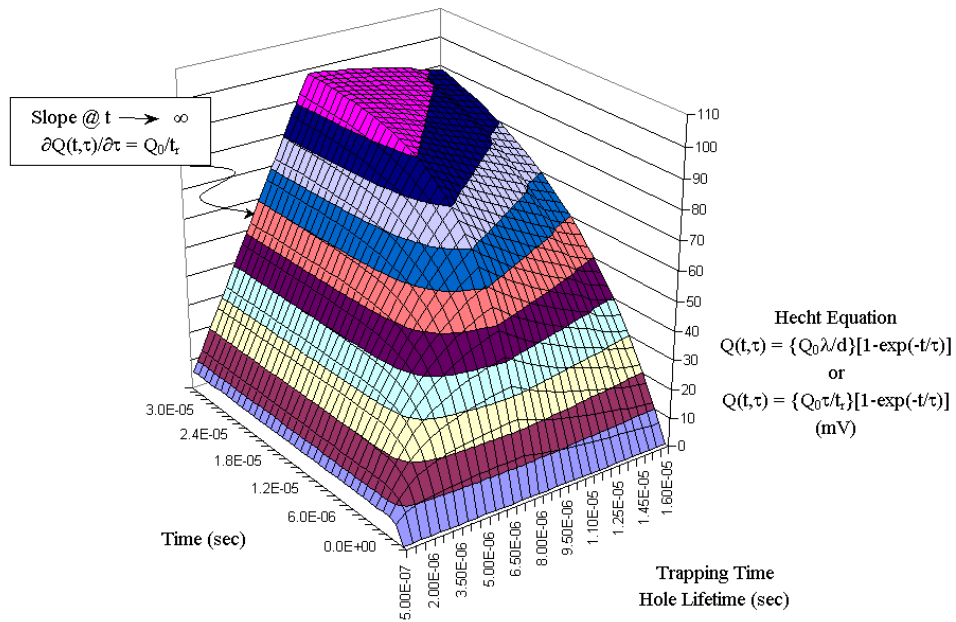


Figure 3. Graph of  $Q_0(\tau/t_r)$  term in Hecht Equation;  $t_r = 2E-7$ .

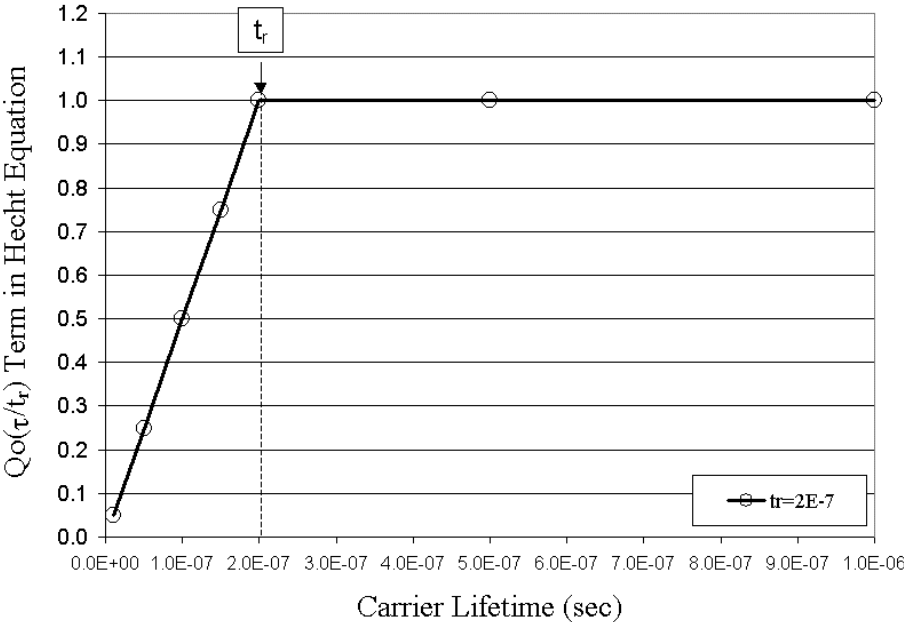


Figure 4. Graph of  $\{1-\exp[-(t/\tau)]\}$  term in the Hecht Equation.

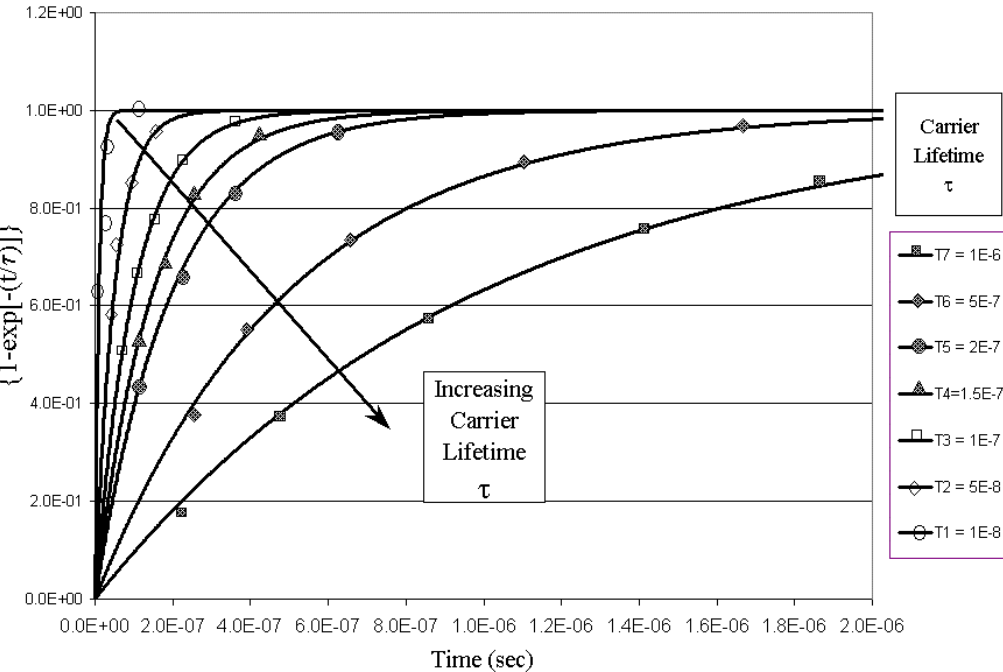


Figure 5. Graph of Hecht Equation.  $Q(t) = Q_0(\tau/t_r)\{1-\exp[-t/\tau]\}$   
Combination of terms from Figures 3 & 4.

